

Acoustic Phononic Crystals with Square-Shaped Scatterers for Two-Dimensional Structures

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Abstract:

Metamaterials are artificial materials that possess unusual physical properties that are not usually found in natural materials. Phononic crystals (PnC) can be constructed by periodic distribution of inclusions embedded in a matrix with high contrast in mechanical properties. They can forbid the propagations of acoustic waves in certain frequencies by creating band gaps. Such band gaps may be independent of the direction of propagation of the incident wave. In present work the acoustic band structure of a two-dimensional phononic crystal consisting of square-shaped rods embedded in air matrix are studied to find the existence of stop bands for the waves of certain energy. The wave band structures of acoustic waves in 2D air/solid phononic structure are investigated theoretically by Finite Element (FE) simulations. A time harmonic analysis of the acoustic wave propagation is performed using the acoustics package of the FE software Comsol Multiphysics v5.3. Phononic band diagrams $\omega=\omega(k)$ for a 2D PnC were plotted versus the wavevector k along the M- Γ -X-M path in the first Brillouin zone. The calculated phonon dispersion results indicate the existence of full acoustic modes in the proposed structure along the high symmetry points.

Key words: Acoustic waves, phononic crystals, metamaterials, band gaps

1. Introduction

There has been a great deal of interest in recent years in the study of two-dimensional (2D) periodic structures, so-called Phononic crystals (PCs) [1–6]. Phononic crystals refer to crystal-like structures that modulate acoustic wave propagation and thus lead to the existence of band gaps in which sound and vibration are all forbidden. The structure is composed of composite materials which are man-made periodically structured material with special properties regarding wave propagation, known as Acoustic Metamaterials [7-9].

The physical properties of phononic crystals can be adjusted artificially by changing the structure of PCs [10]. The choice of materials and their properties as well as the geometrical parameters of PCs have a strong effect on the band structure. The application of phononic crystal strongly depends on the periodicity in three directions in Cartesian coordinates to exploit the band gaps in one-dimensional (1D), two-dimensional (2D) and three-dimensional (3D) structures [11-13]. The band gap characteristics of phononic crystals imply that they can filter elastic or/and acoustic waves in specific frequency ranges. Frequency ranges are deemed to be absolute stop bands if

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any incident wave within the frequency ranges cannot pass through the crystal regardless of its propagation direction. Frequency ranges out of the stop bands are called pass bands. Study of the band structures of phononic crystals paves way to explore other physical properties of these crystals and provides guidance for various applications such as acoustic filters[14,15,16], sound and vibration insulators, ultrasonic silent blocks, and ultrasonic array transducers[17,18].

Different phononic crystal structures have been investigated and corresponding acoustic band gaps have been found for various constituted of materials being both solid, both fluid, and mixed solid/fluid components [19–20]. Complete acoustic band gaps have also been obtained in either squares [21], triangles [22] or honeycomb [23] lattices. Studies of wave propagation along the structures have been usually focused on cylindrical inclusions due to their high symmetry. However, few works suggest that the inclusions have square shaped cross sections arranged in a square lattice or a triangular lattice. Therefore, the present work is motivated by the previous works in phononic crystals [24–26]. Wang et al[27] investigated structures consisted of air rods embedded in dielectric background, and showed that the absolute phononic band gap width is controllable by rotating noncircular scatterers. Li et al [25] investigated the effects of orientations of square rods on the acoustic band gap with the plane-wave expansion (PWE) method. The dispersion relations of square cross section rods in square and triangular lattices forming a PC plate was also studied by the Super Cell method with the equations of the PC plates [26].

The aim of this study is to show the effect of the scatterer shape arranged in a two-dimensional square lattice for a fixed filling ratio. This objective will be realized by investigating the dependence of the gap width on the orientation of the scatterers. More explicitly, this study theoretically and numerically investigates the propagation characteristics and the band structure of longitudinal waves propagating in a 2D phononic crystal composed of LiNbO₃ square rods in air background. We study the 2D scalar acoustic wave propagation in composite materials by solving the basic equations of acoustic wave propagation and use the Bloch theorem for periodic structure to identify the band gaps.

2. Materials and Method

The considered system in this study is a typical 2D phononic crystal composed of LiNbO₃ square inclusions in air background arranged in a square lattice. The calculation model of the studied system, its unit cell, and the rotated square rod by an angle θ are shown in Fig. 1. The material properties are as follows: $\rho_1 = 1.25 \text{ kg/m}^3$ and $c_{LI} = 343 \text{ m/s}$ for air, and $\rho_2 = 4700 \text{ kg/m}^3$ and $c_{LI} = 7430 \text{ m/s}$ for LiNbO₃ with ρ representing density, and c representing velocity of longitudinal waves, respectively. The side length of the square rods is b , and the lattice constant is a .

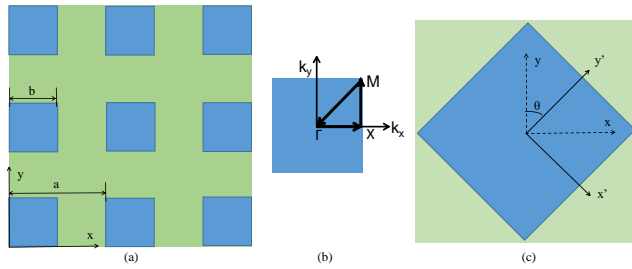


Figure 1. (a) A transverse cross section of two-dimensional square lattice phononic crystal composed of square LiNbO3 inclusions in air, (b) The first Brillouin zone, (c) Rotated square rod with rotation angle θ

Here we focus on the acoustic wave propagation through a periodic array of square scatterers. To calculate the band gaps for phononic crystals, different algorithms have been developed. These numerical methods include, the transfer matrix method (TMM) [28], the finite difference time domain method (FDTD) [29], the multiple scattering theory (MST) [30] method, and the finite element method (FEM) [31].

Different phononic crystal structures have been investigated and corresponding acoustic band. The FEM based on the Bloch theorem is used to calculate the band structure of the proposed PC structure. The governing equation for the acoustic waves is given by frequency-domain Helmholtz equation:

$$\nabla \left(-\frac{1}{\rho} \nabla P \right) = \frac{\omega^2 P}{\rho c_s^2} \quad (1)$$

where p is the acoustic pressure, ρ is the density, ω is the angular frequency, and c_s is the speed of sound. The discrete form of the eigenvalue equations in the unit cell can be written as

$$\left(K^2 - \omega^2 M \right) p = 0 \quad (2)$$

where p is the pressure field at the nodes and K and M can be seen as stiffness and mass matrices of the unit cell, respectively. With the Bloch-Floquet theorem, Bloch periodic boundary conditions were applied on the boundaries of the unit cell:

$$p(\vec{r} + \vec{a}) = p(\vec{r}) e^{-i\vec{k} \cdot \vec{a}} \quad (3)$$

where a is the basis vectors of the lattice and $K=(K_x, K_y)$ is the Bloch wave vector. By varying the value of K along the boundaries of the irreducible first Brillouin zone (BZ) and solving the eigenvalue problem generated by the FEM algorithm, the dispersion relations as well as the eigenmodes of the structure can be obtained.

3. Results and Discussion

With FEM mentioned above, we solved the eigenvalue equation (3) with the FEM software COMSOL Multiphysics 5.3 [32]. The band structure of the PC is calculated with the lattice constant $a=8$ mm and the length of square inclusions $l= 5.6$ mm. First, we investigate the acoustic band structures for different rotation by an angle θ about $\theta =0^\circ$ and $\theta =15^\circ$ at the filling fraction $F=0.49$, respectively. The results are shown in figure 2(a) and (b). There is only one absolute acoustic band gap at $\theta=0^\circ$ in the first fifteen bands, as seen in figure 2(a). In our detailed analysis, numerical calculations show that the structure does not have absolute band gap between the first two bands for any filling fraction at this orientation. However, rotating the square rods the first lowest band gap appear at the rotation angle $\theta=15^\circ$ as shown in figure 2(b). One can see that the degeneracy of the first band at M point is lifted, that opens the lowest band gap among the appearing band gaps.

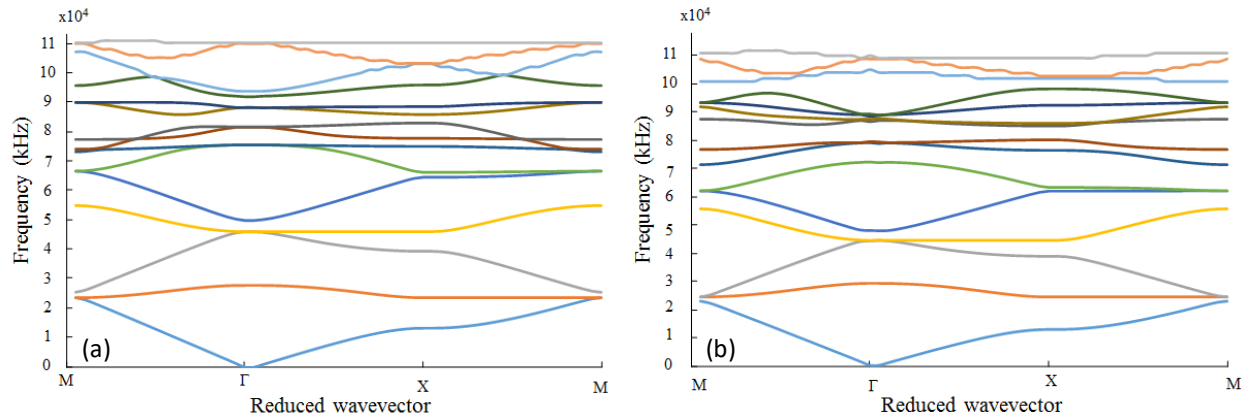


Figure 2. Band structure of PC structure calculated with the FEM. Acoustic band structures for solid rods in air host at filling fraction $F=0.49$, (a) $\theta=0^\circ$, (b) $\theta=15^\circ$

The results show that the gap-width increases progressively with increasing rotation angle. The maximum acoustic band gaps appear at the rotation angle $\theta=45^\circ$ as shown in Figure 3(a). There are six full band gaps at this angle among the first fifteen modes. The band gaps with gap-width from the lowest to the highest band gaps are as follows: $\Delta\omega_1=16.86$ kHz, $\Delta\omega_2=6.47$ kHz, $\Delta\omega_3=9.34$ kHz, $\Delta\omega_4=1.44$ kHz, $\Delta\omega_5=10.61$ kHz and $\Delta\omega_6=4.0$ kHz.

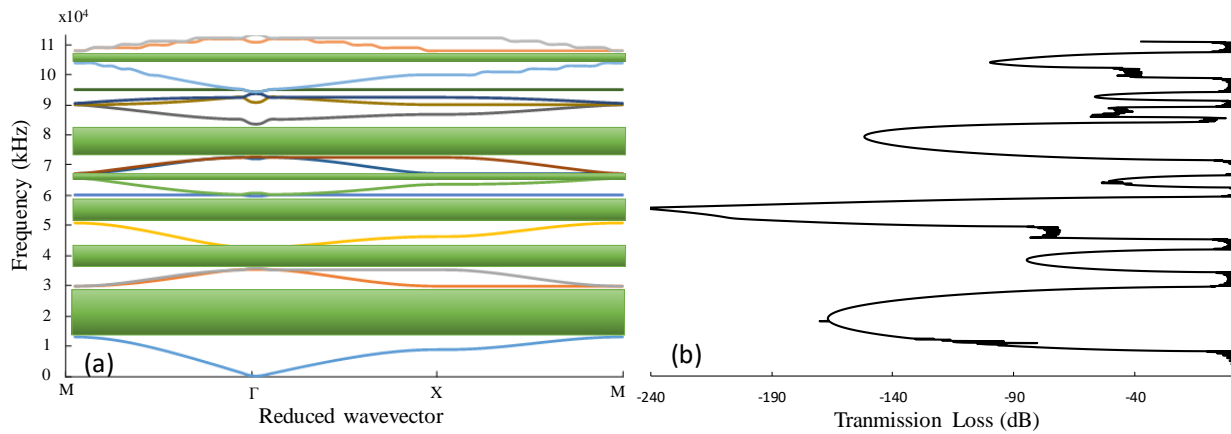


Figure 3. Comsol computed band structure in comparison to the simulated transmission measurements

Finite Element (FE) simulations are utilized in order to model the acoustic wave propagation and scattering through the suggested phononic crystal. Analysis of time harmonic propagation of acoustic wave is performed using the acoustic module of the FE software Comsol Multiphysics 5.3. The frequencies at which the band gaps occur in the band structure are in good agreement with the regions of attenuation present in the transmission loss spectrum, Figure 3(b). The transmission coefficient for those particular frequency segments are near zero and therefore corresponds to the blank regions along the ΓX direction of the band structures. To study the propagation properties of acoustic waves in the PC structure, the maps of pressure fields of the PC structure at different frequencies are analyzed. A finite structure composed of 5×10 units is modeled for the calculation. The same model is also used for transmission loss spectrum.

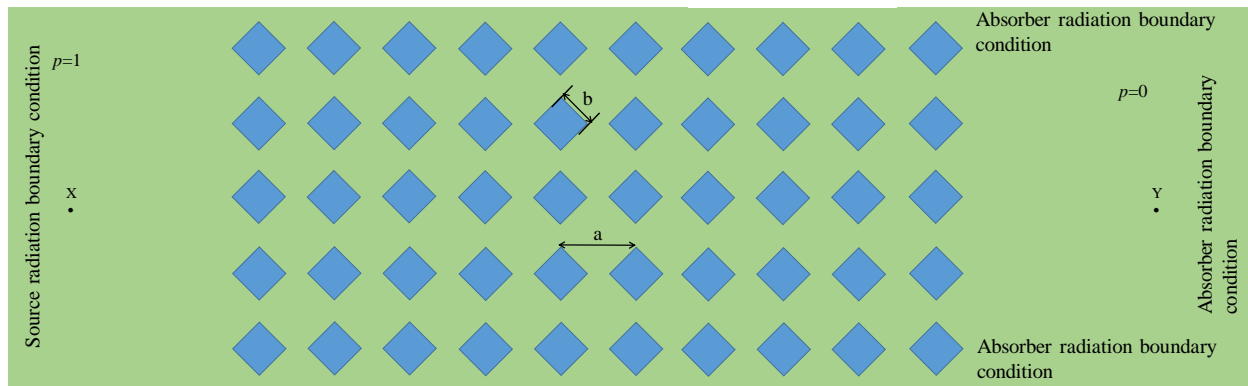


Figure 4. Comsol described geometry for analysis of two-dimensional time harmonic wave. The Theoretical measurements were taken at two points marked as X and Y.

Parametric Sweep spanning the wave was used to calculate the frequency spectrum for wave propagation through the phononic crystal system. Figure 5 shows the map of pressure field at the frequency $f=20$ kHz, which is inside the lowest band gap. From Figure 5 it can be observed that,

with the acoustic wave propagating from left to right in the structure, the acoustic pressure gradually decreases and becomes nearly zero when the waves reach the fifth column.

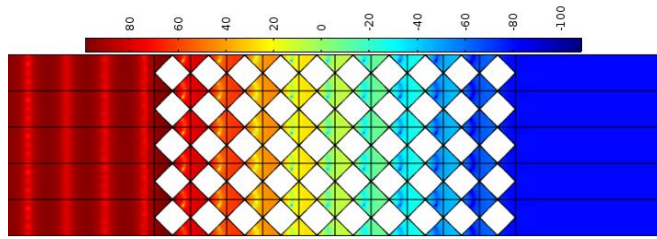


Figure 5. Map of the sound pressure fields in the PC composed of 5×10 units at frequency at $f=20$ kHz

Figures 6(a) and 6(b) show the dispersion surfaces of the 2D PnC system for the first and third modes. The difference in dispersion curves in Figure 6(a) and 6(b) is very clear. However, these dispersion surfaces carry important information because each point on the surface gives the possible Eigen solutions that consist of all the allowed wave vectors in the first Brillouin zone. It can be deduced that dispersion curves may be useful for quantitative information. Therefore, isotropic or anisotropic wave propagation of waves in PnC can be best analyzed by the equal frequency contour (EFC).

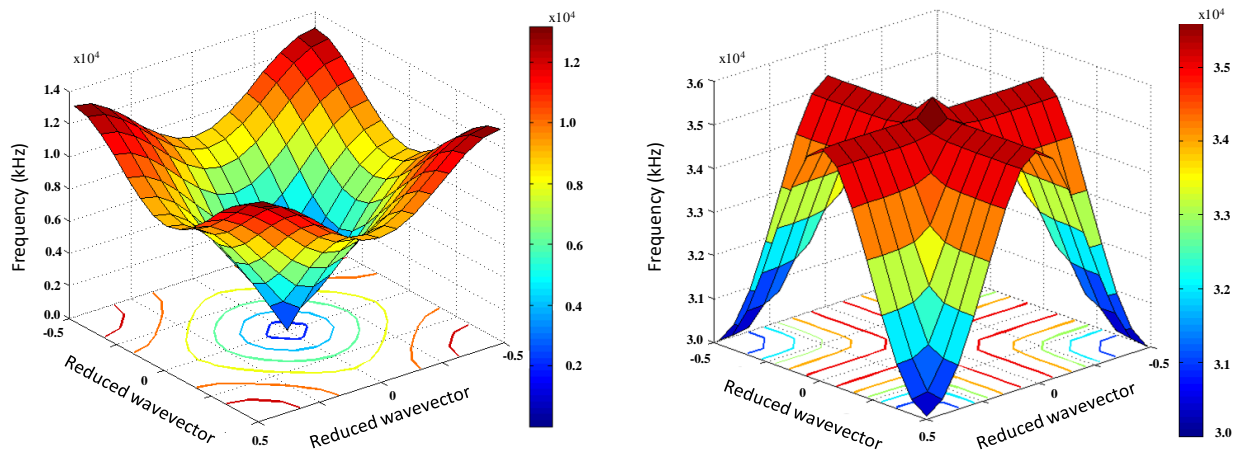


Figure 6. The dispersion, $\omega(k)$, relation (band-diagram) for the first bands of the square phononic crystal., calculated for all k -vectors in the first Brillouin zone

Conclusions

In summary, the existence of complete band gap was theoretically investigated in a 2D phononic crystal consisting of a square array of LiNbO₃ square rods in air. It is found that the band gap can be controlled by changing the scatterer orientation and the appearance of the bands increases as a result of scatterer rotation. We show the effect of the noncircular shape arranged in a two-dimensional square lattice for a fixed filling ratio. Numerical results also show that the transmission loss spectrum calculated with the model is in good agreement with the dispersion diagram.

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